

# WEAK DIPOLE MOMENTS AT $e^+e^-$ COLLIDERS\*

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## ABSTRACT

The weak dipole moments of leptons and quarks, *i.e.* those related to their  $Z$ -coupling, are reviewed. Standard Model predictions and experimental results may result in a stringent test for both their pointlike structure and also for the Standard Model. Special attention is devoted to the anomalous weak-magnetic dipole moment and to the  $CP$ -violating weak-electric dipole moment.

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# 1 Introduction

The dipole moments of the electron and muon provide very precise tests of quantum electrodynamics. The prediction obtained for the first time by Schwinger [1] was one of the most spectacular achievements of quantum field theories. With the advent of LEP, an enormous variety of measurements lead to the confirmation of the Standard Model. Nowadays, even pure weak–quantum corrections are been tested at one loop. The Standard Model predictions for the cross sections, widths and various asymmetries have been successfully confronted with measurements.

In this review article we will concentrate on tests and predictions, coming from the Standard Model, on the weak–dipole moments: weak–magnetic and weak–electric ones. Both weak–magnetic and weak–electric effective lagrangean terms have the same chirality flipping structure as mass terms. They receive contributions from electroweak radiative corrections in the Standard Model, but also new physics contributions may show up in them. In particular, they may provide insight into the origin of mass.

Dipole moments are quantum corrections to the tree level matrix elements. Within the Standard Model, whereas the anomalous weak–magnetic dipole moment (WMDM) receives its leading value from one loop corrections, the case for the weak–electric (WEDM) is rather different. Being a  $CP$ –violating property, it receives a non–vanishing value through the Kobayashi–Maskawa mechanism, and thereby only at very high order in the coupling constant.

To study these dipole moments, two different approaches are possible. First of all, one can compute the Standard Model predictions for them. The other approach is the effective lagrangean approach. In this case the dipole moments arise as low energy contributions from a high energy physics scale. The WEDM will be sensitive to new physics, whereas the WMDM will receive contributions from electroweak radiative corrections too. The corresponding operators in the effective lagrangean are dimension–5 operators with the normalization to  $g/(2m_f)$ ; Marciano [2] has argued the very general result that a fermion of mass  $m_f$ , generated at  $\Lambda$ –scale, has an

anomalous moment  $\text{WDM} \sim m_f^2/\Lambda^2$ .

In both approaches one should try to construct sensitive observables to them: as any radiative correction they contribute to many observables, but one should identify the appropriate ones in order to disentangle them. In some observables, for example, like cross sections and widths, these dipole moments are hidden by tree level contributions and also mixed up with other quantum corrections. One has to deal with the spin properties in order to obtain sensitive observables. The spin density matrix of the produced lepton/quark pairs has sensitive terms to the dipole moments, both in the single lepton/quark-polarization and in the spin-spin correlation terms. The discrete symmetry properties of the dipole moments allow a clear search out of these observables. For example, in order to disentangle the WMDM, the parity-odd and time reversal-even single lepton transverse polarization (within the collision plane) is found to be a good candidate [3, 4]. The  $CP$ -violating WEDM can be found in the single polarization terms and in correlation terms of the spin density matrix. Genuine (*i.e.* the ones that do not receive contributions from unitarity corrections and are non-vanishing only if  $CP$ -violating pieces appear in the lagrangean)  $CP$ -violating observables can be defined from both single polarization and from correlations terms, but also non genuine  $CP$ -violating observables may be useful in order to put bounds on these dipole moments. This paper is organized as follows: in section 2 the results appearing in the literature related to the WMDM are reviewed, in section 3 the WEDM for leptons is studied, and in section 4 it is shown how the WEDM for quarks, mainly the top quark, can be bounded. Section 5 presents some conclusions and the perspectives on this topic.

## 2 Weak-Magnetic Dipole Moment

The anomalous WMDM is generated through a chirality flip mechanism, so it is expected to be proportional to the mass of the particle. Thereby, only heavy leptons and quark are good candidates to have a sizable WMDM:  $\tau$ ,  $\mathbf{c}$ ,  $\mathbf{b}$  and  $\mathbf{t}$  quarks.

The weak-magnetic dipole moment was investigated for the  $\tau$  in [3, 4], and the

WMDM for  $b$  is studied in [7]. The electroweak gauge invariant anomalous WMDM for the  $\tau$  was computed in [4] within the Standard Model, to first order in the coupling constant, and appropriate observables to measure it were studied in [3, 4]. There, it is shown that for  $e^+ e^- \longrightarrow \tau^+ \tau^-$  unpolarized scattering at the  $Z$ -peak, the transverse (within the collision plane) and normal (to the collision plane) single  $\tau$  polarizations are sensitive to the real and imaginary parts of the anomalous weak-magnetic dipole moment, respectively. Polarization measurements are accessible for the  $\tau$  by means of the energy and angular distribution of its decay products.

The WMDM is defined in the following way. The matrix element of the vector neutral current coupled to the  $Z$  is written, using Lorentz covariance, in the form

$$\bar{u}(p_-) V^\mu(p_-, p_+) v(p_+) = e \bar{u}(p_-) \left[ \frac{v(q^2) \gamma^\mu}{2s_w c_w} + i \frac{a_\tau^w(q^2)}{2m_\tau} \sigma^{\mu\eta} q_\eta \right] v(p_+) \quad (1)$$

where  $q = p_- + p_+$ ,  $e$  is the proton charge and  $s_w, c_w$  are the weak mixing angle sine and cosine, respectively. The first term  $v(q^2)$  is the Dirac vertex (or charge radius) form factor and it is present at tree level with a value  $v(q^2) = \frac{1}{2} - 2s_w^2$ , whereas the second form factor is the WMDM and only appears due to quantum corrections. Only the on-shell vertex with  $q^2 = M_Z^2$  is entitled to be electroweak gauge invariant in the Standard Model.

In order to compute the anomalous WMDM, there are 14 diagrams to calculate in the t'Hooft-Feynman gauge. In Figure 1 a generic diagram is shown;  $\alpha, \beta, \gamma$  stand for the particles circulating in the loop:  $N\tau^+\tau^-, \nu C^+C^-, \nu\nu C^-, \tau^- NN'$ , where  $N, N' = \gamma, Z, \chi, \Phi$ ,  $N \neq N'$  and  $C = W^\pm, \sigma^\pm$  are all the diagrams present in the calculus. We denote by  $\sigma^\pm$  the charged non-physical Higgs and by  $\chi$  and  $\Phi$  the neutral non-physical and physical ones.

There are 6 diagrams that are not present in the analogous photon vertex case. these have the following particles circulating in the loop:  $W\nu\nu, \sigma^-\nu\nu, \tau Z\Phi, \tau\Phi Z, \tau\Phi\chi, \tau\chi\Phi$ . In fact, one of these (the one with  $W\nu\nu$  in the loop) gives the leading contribution; this show that the quantity is governed by quantum pure-weak effects.

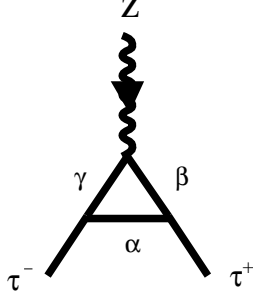


Figure 1: Contributing Feynman diagrams to the WMDM in the t'Hooft–Feynman gauge.

All contributions can be written as:

$$a_{ABC} = \frac{\alpha}{4\pi} \frac{m_\tau^2}{M_Z^2} \sum_{ij} c_{ij} I_{ij}^{ABC} \quad (2)$$

where  $A$ ,  $B$  and  $C$  are the particles circulating in the loop, counting clockwise in the diagrams from the particle between the two fermion lines,  $c_{ij}$  are coefficients depending on masses and coupling constants, and  $I_{ij}^{ABC} \equiv I_{ij}(m_\tau^2, q^2, m_\tau^2, m_A^2, m_B^2, m_C^2)$  are scalar, vector or tensor 3-point functions defined in [4].

When computing the diagrams we only select the tensor structure related to the WMDM, and we also verify the vector current conservation as a check of our expressions (there is no induced  $(p_- + p_+)^\mu$  term in Eq.(1)). The external lines are on the mass shell, *i.e.*  $p_-^2 = m_\tau^2$ ,  $p_+^2 = m_\tau^2$  and  $(p_- + p_+)^2 = M_Z^2$  respectively. Some of the diagrams with the propagation of Higgs or would-be Goldstone bosons particles are suppressed by extra  $(\frac{m_\tau^2}{M_{Z,\Phi}^2})$  terms in such a way that the  $a_{\chi\tau\tau}$ ,  $a_{\Phi\tau\tau}$ ,  $a_{\sigma\nu\nu}$  and  $a_{\sigma\sigma\nu}$  contributions to  $a_\tau^w$  are negligible. Diagrams in which the Higgs and the neutral would-be Goldstone boson particles couple to the  $Z$  only contribute to the axial form factor and not to the magnetic moment ( $a_{\tau\Phi\chi} = a_{\tau\chi\Phi} = 0$ ).

The  $I_{ij}^{ABC}$  functions were analytically computed in terms of dilogarithm functions, and checked with a numerical integration for the  $m_\tau \rightarrow 0$  limit. Some details

are given in [4]. They obtain that the numerical contribution of each diagram is:

$$\begin{aligned}
a_{\gamma\tau\tau} &= -\frac{\alpha}{4\pi} \frac{m_\tau^2}{M_Z^2} (1.32 - 0.52i) \simeq (3.12 - 1.23i) \times 10^{-7} \\
a_{Z\tau\tau} &= \frac{\alpha}{4\pi} \frac{m_\tau^2}{M_Z^2} (0.17 + 0.08i) \simeq (3.92 + 1.88i) \times 10^{-8} \\
a_{\nu WW} &= \frac{\alpha}{4\pi} \frac{m_\tau^2}{M_Z^2} (-7.07) \simeq -1.68 \times 10^{-6} \\
a_{\nu W\sigma} &= \frac{\alpha}{4\pi} \frac{m_\tau^2}{M_Z^2} 0.45 \simeq 1.06 \times 10^{-7} \\
a_{\nu\sigma W} &= \frac{\alpha}{4\pi} \frac{m_\tau^2}{M_Z^2} 0.45 \simeq 1.06 \times 10^{-7} \\
a_{\tau\Phi Z} = a_{\tau Z \Phi} &= -\frac{\alpha}{4\pi} \frac{m_\tau^2}{M_Z^2} (0.07 ; 0.03 ; 0.02) \simeq -(0.15 ; 0.07 ; 0.04) \times 10^{-7} \\
a_{W\nu\nu} &= \frac{\alpha}{4\pi} \frac{m_\tau^2}{M_Z^2} (-4.11 - 2.12i) \simeq -(0.974 + 0.502i) \times 10^{-6} \tag{3}
\end{aligned}$$

where the values between parenthesis for  $a_{\tau\Phi Z} = a_{\tau Z \Phi}$  correspond to  $\frac{M_\Phi}{M_Z} = 1, 2, 3$  respectively.

Finally, the value of the computed WMDM is

$$a_\tau^w(M_Z^2) = -(2.10 + 0.61i) \times 10^{-6} \tag{4}$$

The Higgs mass only modifies the real part of this result less than a 1%, from the value  $-2.12 \times 10^{-6}$  to  $-2.10 \times 10^{-6}$  for  $1 < \frac{M_\Phi}{M_Z} < 3$ . In Eq.(4) we have chosen  $M_\Phi = 2M_Z$ .

We should point out that, contrary to the well known photon–electroweak anomalous magnetic moment, the non–vanishing absorptive part in Eq.(4) is due to the fact that we compute on the  $Z$  mass shell  $q^2 = M_Z^2$ , not  $q^2 = 0$ . In fact, one expects a non–vanishing imaginary part coming from unitarity.

High–precision measurements at LEP/SLC were used in [5] in order to extract bounds for the weak–dipole moments. They found bounds of the order  $10^{-1} - 10^{-2}$  for

the WMDM of  $\tau, c$  and  $b$  quarks, where data from the  $Z$ -width, forward-backward asymmetry and additional angular distributions were used. However, one should take into account that these observables are not the most appropriate ones in order to extract this kind of information. For one hand they receive contributions from any radiative correction or new physics terms, but on the other, they do not depend linearly on the dipole moments. For example, one should identify observables that vanish when the fermion mass (and the dipole moments) vanishes: these are, in principle, good candidates in order to measure the dipole moments. For  $e^+e^- \rightarrow \tau^+\tau^-$  unpolarized collisions, in the  $m_f = 0$  limit the only non-vanishing component of  $\tau$ -polarization is the longitudinal one. Then, both the transverse (within the collision plane) and normal (to the collision plane) polarization components vanish in that limit. These ideas were developed in [3, 4]: when  $m_f, a_f^w \neq 0$  then  $P_N^f, P_T^f \neq 0$ . The transverse and normal single  $\tau$  polarization terms were used to construct asymmetries proportional to these dipole-moments. The transverse polarization term in the cross section is proportional to the real part of the WMDM, except for a small helicity-flip suppressed tree level contribution:

$$\left. \frac{d\sigma}{d\Omega_{\tau^-}} \right|_T = \frac{\alpha^2 \beta}{128 s_w^3 c_w^3 \Gamma_Z^2} a \sin \theta_{\tau^-} \left\{ - \left[ 2v^2 + (v^2 + a^2) \beta \cos \theta_{\tau^-} \right] \frac{v}{\gamma s_w c_w} + 2\gamma \left[ 2v^2(2 - \beta^2) + (v^2 + a^2) \beta \cos \theta_{\tau^-} \right] \text{Re}(a_\tau^w) \right\} (s_- + s_+)_x \quad (5)$$

The normal polarization term is proportional to the absorptive part of the WMDM, except for possible WEDM or electric dipole moment contributions:

$$\left. \frac{d\sigma}{d\Omega_{\tau^-}} \right|_N = \frac{\alpha^2 \beta}{128 s_w^3 c_w^3 \Gamma_Z^2} (-2v) \gamma \beta \sin \theta_{\tau^-} \left[ 2a^2 + (v^2 + a^2) \beta \cos \theta_{\tau^-} \right] \times \text{Im}(a_\tau^w) (s_- + s_+)_y \quad (6)$$

where  $a$  and  $v$  are the neutral axial and vector couplings,  $\gamma = \frac{m_Z}{2m_\tau}$  is the dilation factor,  $\beta$  is the  $\tau$  velocity and  $\theta_{\tau^-}$  is the angle determined by the  $e^-$  and the  $\tau^-$  momenta.

The spin properties of the  $\tau$  can only be analyzed from their decay products. In order to have access to the single  $\tau$  polarization, the  $\tau$  frame has to be reconstructed.

Micro-vertex detectors allow such a reconstruction, as was shown in [6], for the case in which both  $\tau$ 's decay into (at least one) hadrons and their energies and tracks are reconstructed.

Let us consider the processes  $e^+e^- \longrightarrow \tau^+\tau^- \longrightarrow h_1^+ X h_2^- \nu_\tau$  and  $e^+e^- \longrightarrow \tau^+\tau^- \longrightarrow h_1^+ \bar{\nu}_\tau h_2^- X$ . One can construct a sort of mixed “up-down-forward-backward” asymmetries in order to disentangle the dispersive and absorptive parts of the WMDM. They select the leading  $\cos \theta_\tau \cos \Phi_{h^\mp}$  and  $\cos \theta_\tau \sin \Phi_{h^\mp}$ , respectively, in the cross section:

$$A_{Dis}^\mp = \frac{\sigma_{Dis}^\mp(+)-\sigma_{Dis}^\mp(-)}{\sigma_{Dis}^\mp(+)+\sigma_{Dis}^\mp(-)} \quad (7)$$

with

$$\sigma_{Dis}^\mp(\pm) = \left[ \int_0^1 d(\cos \theta_{\tau^-}) \int_{-\pi/2(\pi/2)}^{\pi/2(\frac{3}{2}\pi)} d\phi_{h^\mp} + \int_{-1}^0 d(\cos \theta_{\tau^-}) \int_{\pi/2(-\pi/2)}^{\frac{3}{2}\pi(\pi/2)} d\phi_{h^\mp} \right] \times \frac{d\sigma}{d(\cos \theta_{\tau^-}) d\phi_{h^\mp}} \quad (8)$$

and

$$A_{Abs}^\mp = \frac{\sigma_{Abs}^\mp(+)-\sigma_{Abs}^\mp(-)}{\sigma_{Abs}^\mp(+)+\sigma_{Abs}^\mp(-)} \quad (9)$$

where

$$\sigma_{Abs}^\mp(\pm) = \left[ \int_0^1 d(\cos \theta_{\tau^-}) \int_{0(\pi)}^{\pi(2\pi)} d\phi_{h^\mp} + \int_{-1}^0 d(\cos \theta_{\tau^-}) \int_{\pi(0)}^{2\pi(\pi)} d\phi_{h^\mp} \right] \times \frac{d\sigma}{d(\cos \theta_{\tau^-}) d\phi_{h^\mp}} \quad (10)$$

After some algebra one finds:

$$A_{Dis}^\mp = \mp \alpha_h \frac{s_w c_w}{4\beta} \frac{v^2 + a^2}{a^3} \left[ -\frac{v}{\gamma s_w c_w} + 2\gamma \operatorname{Re}(a_\tau^w) \right] \quad (11)$$

$$A_{Abs}^\mp = \mp \alpha_h \frac{3\pi\gamma}{4} c_w s_w \frac{v}{a^2} \operatorname{Im}(a_\tau^w) \quad (12)$$

where  $\alpha_h$  is a parameter that expresses the sensitivity of each channel to the  $\tau$ -spin properties. The  $\mp$  signs refer to the processes defined above. Collecting events from the  $\pi, \rho$  and  $a_1$  channels it is possible to put the following bounds:

$$|\operatorname{Re}(a_\tau^w)| \leq 4 \cdot 10^{-4} \quad (13)$$



$$|Im(a_\tau^w)| \leq 1.1 \times 10^{-3} \quad (14)$$

These results were obtained considering  $10^7$   $Z$  events, and the semileptonic decay channels considered amount to about 52% of the total decay rate.

The Standard Model predictions for the real and imaginary parts of the WMDM are not actually accessible nowadays. Any signal coming from the observables defined above should be related to new physics.

### 3 Weak–Electric Dipole Moment

In this section the experimental status and the theoretical results concerning the WEDM for leptons are presented. Let us begin by the theoretical expectations. The time reversal–odd WEDM depends on the underlying physics of the CP violation mechanisms of the model. Standard Model  $CP$ –violating effects, as the WEDM for leptons should, in principle, not be observable within the present experimental limits: they receive contributions through the Kobayashi–Maskawa matrix and only at very high order in the coupling constant. This is the main reason to look after them: a non–vanishing signal related to them would be a clear claim for physics beyond the Standard Model. In many extensions of the Standard Model and the Kobayashi–Maskawa mechanism, the  $CP$ –violation in the lepton sector occurs naturally, and the generated electric and weak–electric dipole moments are proportional to the mass of the lepton. At the  $Z$ –mass scale, it is therefore sensible to investigate the heavy flavours. A large number of papers are devoted to the study of the  $\tau$  dipole moments, and we will now review their paramount results.

The subject of  $CP$ –violation and its related observables has received much attention in recent years. Both  $\tau$  and  $t$  have a large branching ratio in weak decays, so  $\tau$  and  $t$  physics may be studied in a similar way. Theoretical work done for the tau can be extended so as to be useful for the top. Some authors have investigated the Lorentz structure of the  $\tau - \tau - Z$  vertex, looking for test of discrete symmetries and possible Standard Model deviations. In refs.[8, 9, 10] some of the results on  $\tau$  physics

were studied. More references can be found in these articles. The WEDM is defined in the following way. The matrix element of the axial–vector neutral current coupled to the  $Z$  is written, using Lorentz covariance, in the form

$$\bar{u}(p_-) A^\mu(p_-, p_+) v(p_+) = e \bar{u}(p_-) \left[ \frac{a(q^2) \gamma^\mu \gamma_5}{2s_w c_w} + \frac{1}{e} d_\tau^w(q^2) \sigma^{\mu\eta} \gamma_5 q_\eta \right] v(p_+) \quad (15)$$

Again, the first term  $a(q^2)$  is the Dirac axial vertex form factor and it is present at tree level with a value given by the third component of weak–isospin, whereas the second form factor is the WEDM and only appears due to quantum corrections at very high orders. In extended models with scale  $\Lambda$ , one expects  $d_f^w \sim m_f/\Lambda^2$ .

A sizable WEDM would lead to a deviation (proportional to the square of the WEDM) of the cross section for  $e^+e^- \rightarrow f\bar{f}$  from its standard value. In this way Barr and Marciano [11] have taken into account the PETRA results for the cross section to deduce bounds for the electric dipole moment of the tau–lepton.

A WEDM induces [9] an additional contribution to the  $Z$  partial width:

$$\Delta\Gamma_\tau = |d_\tau^w|^2 \frac{m_Z^3}{24\pi} \quad (16)$$

From the comparison of the value measured [12] at LEP of the  $Z$  partial width  $\Gamma_\tau = (84.26 \pm 0.34) \text{ MeV}$  and the Standard Model theoretical prediction [13]  $\Gamma_\tau^{SM} = (83.7 \pm 0.4) \text{ MeV}$  an upper limit

$$|d_\tau^w| \leq 2.3 \times 10^{-17} e \text{ cm} \quad (17)$$

at 95% C.L. is obtained.

This argument is an indirect one, and other  $CP$ –even and  $CP$ –odd effects coming from other terms, for example new physics, may compete. Moreover, this is certainly not the most efficient way to put bound on these dipole moments; for instance, linear effects in the dipole moment (through  $CP$ –odd observables) allow to put stringent limits. One should first understand where the most important effects coming from these dipole moments manifest themselves and then, one should either look for genuine  $CP$ –violating observables or to look for observables where effects coming from non  $CP$ –violating pieces of the lagrangean are suppressed.

At the  $Z$ -peak the electric and weak-electric dipole moments can be separated in the observables: the last one should be enhanced at his natural scale, while the first one is suppressed.

All the information one can extract from the process  $e^+e^- \longrightarrow \tau^-\tau^+$  is contained in the spin density matrix. It has terms that allows to define genuine and non-genuine  $CP$ -violating observables. In [10] the first alternative was chosen, whereas in [3] the second one was chosen.

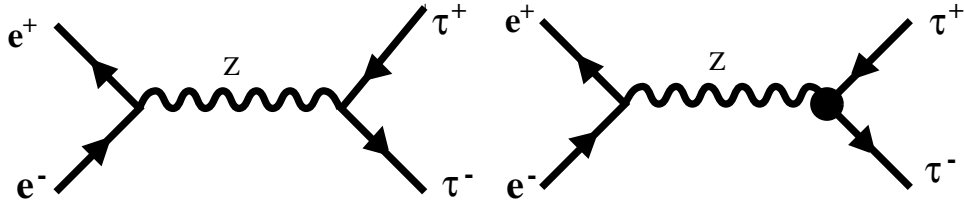


Figure 2: Leading Feynman diagrams considered for processes related to the WEDM: the left is the tree level diagram while the right one is the leading one in the  $CP$ -violating effective vertex.

In [10] triple correlation products of momenta were used to define genuine  $CP$ -violating observables in  $\tau^\pm$  pair production. The  $\tau$  lepton decays before reaching the detectors, and only momenta of the  $\tau$  decay products are, in principle, accessible. This triple correlation products are generated by the spin-spin correlation terms in the spin density matrix. For example, for the case where both  $\tau$ 's decay into  $\pi\nu$ , the knowledge of the  $\hat{q}_\pm = \frac{q_\pm}{|q_\pm|}$  momenta of both  $\pi^\pm$  allow to compute the centre of mass expectation value of the following tensor observables

$$\begin{aligned} T_{ij} &= (q_+ - q_-)_i (q_+ \times q_-)_j + (i \leftrightarrow j) \\ \hat{T}_{ij} &= (\hat{q}_+ - \hat{q}_-)_i \frac{(\hat{q}_+ \times \hat{q}_-)_j}{|\hat{q}_+ \times \hat{q}_-|} + (i \leftrightarrow j) \end{aligned} \quad (18)$$

The expectation value is proportional to the WEDM:

$$\langle T_{ij} \rangle_{AB} \simeq \frac{m_Z}{e} c_{AB} s_{ij} d_\tau^w \quad (19)$$

$$\langle \hat{T}_{ij} \rangle_{AB} \simeq \frac{m_Z}{e} \hat{c}_{AB} s_{ij} d_\tau^w \quad (20)$$

where A and B are the decay modes of  $\tau^-$  and  $\tau^+$ , respectively. The constants  $c_{AB}$  and  $\hat{c}_{AB}$  describe the sensitivity of the different decay channels to the spin properties of the  $\tau$ , and  $s_{ij}$  is the tensor polarization of the  $Z$ . One can construct  $CP$ -odd tensor observables and that are time reversal even or odd. In this way the absorptive and dispersive parts of the WEDM can be tested. Taking into account only the Standard Model amplitudes and the first order WEDM effective vertex (shown in Figure 2) these tensor observables yield the following sensitivities. At the  $Z$ -peak, and with  $10^7$   $Z$  events an upper limit  $1.3 \times 10^{-18} \text{ e cm}$  for the dispersive part of the WEDM is claimed, whereas for the absorptive one the limit is  $3.2 \times 10^{-17} \text{ e cm}$ .

These ideas were followed by the OPAL [14] and ALEPH [15] Collaborations to put bounds on the dispersive and absorptive parts of the WEDM. Collecting 20.000  $\tau^\pm$  events from 1990–1992 data, and studying the decay channels  $e\nu\nu, \mu\nu\nu, \pi\nu, \rho\nu, a_1\nu$ , ALEPH found, at the 95% C.L., the upper bound

$$|d_\tau^w| \leq 1.5 \times 10^{-17} \text{ e cm} \quad (21)$$

Data from 1991-1993 were used by OPAL, altogether resulting in 28000  $\tau^\pm$  pairs to find:

$$|Re(d_\tau^w)| \leq 7.8 \times 10^{-18} \text{ e cm} \quad (22)$$

$$|Im(d_\tau^w)| \leq 4.5 \times 10^{-17} \text{ e cm} \quad (23)$$

at 95% C.L., where the decay channels  $ll', lm$  and  $mm'$  where  $l, l' = e\mu$  and  $m, m' = \pi, \rho, a_1$  were taken into account (except for the  $a_1 a_1$  decay channel) in order to obtain the first result, and the channels  $ee, \mu\mu, \pi\pi, \pi\rho, \rho\rho$  were used in the second one.

Similar tensor observables may also be useful when the initial state spin density matrix is not  $CP$ -even. They cease to be genuine  $CP$ -violating observables. In [16] the  $e^-$  beam was considered with longitudinal polarization, and they argue that  $CP$ -even (suppressed by the electron mass) and  $CP$ -odd (suppressed by a factor  $\alpha^2 \Gamma_Z^2 / m_Z^2$ ) effects coming from the initial state can be discarded. In this case the  $CP$ -odd,  $P$ -odd correlations (19) and (20) are not necessarily proportional to the

small parity violating parameter  $r = 2v^e a^e / ((v^e)^2 + (a^e)^2)$  in the electron vertex, and  $r$  is replaced by the much higher longitudinal polarization  $P_L^e$  of the beam, about 70% at SLC. A sensitivity of  $10^{-17} \text{ } e \text{ cm}$  would be achieved when  $\pi\nu$  and  $\rho\nu$  decay channels for  $10^6$   $Z$  events with  $e^-$  polarization 62-75 % (likely to be available at the SLC at Stanford) is supposed.

In [3] the single  $\tau$  polarization pieces of the spin density matrix were used in order to define observables sensitive to the WEDM of the tau-lepton. The normal polarization of a single  $\tau$  is parity-even and time reversal-odd, and although it is not a genuine  $CP$ -violating quantity it enjoys the following virtues:

- i) it gets a contribution from  $CP$ -conserving interactions only through the combined effect of both an helicity-flip transition and the presence of absorptive parts (unitarity corrections), which are both suppressed in the Standard Model,
- ii) with a  $CP$ -violating interaction such as a WEDM, it gets a non-vanishing value without the need of absorptive parts,
- iii) as the leading observable effect comes from the interference of the  $CP$ -violating amplitude with the standard amplitude and the observable is P-even, the sensitivity of the normal polarization to linear terms in the (dispersive) WEDM is enhanced by the leptonic axial neutral current Standard coupling and no need of the suppressed vector coupling of the  $Z$  to  $\tau$  appears.

One has still the possibility to compare the normal polarization for  $\tau^+$  and  $\tau^-$ , thus obtaining a true  $CP$ -violating observable but with the half of statistics. The single  $\tau$  normal polarization for the process is proportional to:

$$2a\gamma\beta \sin\theta_{\tau-} \left[ 2v^2 + (v^2 + a^2)\beta \cos\theta_{\tau-} \right] \text{Re}(d_\tau^w) \quad (24)$$

In order to extract  $CP$ -violating information from the normal polarization it is necessary to reconstruct the  $\tau$  direction. This possibility was studied in [6], and (24) appears in the  $\sin\theta_\tau \cos\theta_\tau \sin\Phi_h$  distribution of the decay products. It is then possible to construct asymmetries sensitive to the WEDM, as:

$$A^\mp = \frac{\sigma^\mp(+)-\sigma^\mp(-)}{\sigma^\mp(+)+\sigma^\mp(-)} \quad (25)$$

where

$$\sigma^\mp(\pm) = \frac{\left[ \int_0^1 d(\cos \theta_{\tau^-}) \int_{0(\pi)}^{\pi(2\pi)} d\phi_{h^\mp} + \int_{-1}^0 d(\cos \theta_{\tau^-}) \int_{\pi(0)}^{2\pi(\pi)} d\phi_{h^\mp} \right]}{d\sigma} \times \quad (26)$$

One gets, in the presence of a non-vanishing  $d_\tau^w$ , for both  $\tau^\pm$ :

$$A^\mp = \alpha_h \frac{\gamma}{2} s_w c_w \frac{v^2 + a^2}{a^3} \text{Re}(d_\tau^w) \quad (27)$$

$$A^{CP} \equiv \frac{1}{2}(A^- + A^+) \quad (28)$$

Although  $A^\mp \neq 0$  is not a genuine CP-odd term,  $A^{CP} \neq 0$  does. The factor  $\alpha_h$  describe the sensitivity of the decay channel with the hadron  $h$  to the spin properties of the  $\tau$ . The  $A^\pm$  asymmetries may receive contribution from helicity-flipping transitions coming from unitary corrections, as it was anticipated in i). These contributions are negligible and not show in the above expression. In fact, the absorptive part of the WMDM is one of them. The upper indexes  $\pm$  denote that we are collecting events for the  $\mp \tau$  decay channel and that the asymmetry is constructed using the  $\tau^\pm$  decay product. What is tested in the second asymmetry is whether the normal polarization of both taus are different, and this is certainly a genuine test of  $CP$ -violation. Following these ideas, a sensitivity

$$|d_\tau^w| \leq 2.3 \times 10^{-18} \text{ e cm} \quad (29)$$

to the WEDM is found.

## 4 Dipole Moments for Quarks

The evidence for the top quark existence has generated much excitement and a large amount of theoretical work is nowadays devoted to it. All kinds of possible tests of its properties, in particular possible extensions to new physics are being currently investigated.  $CP$ -violation related to  $t\bar{t}$  production has received much attention. In this section we will review some of these results, but mainly connected to the dipole

moments of the top. Standard Model predictions and the search for possible new physics effects in top quark production and decay look promising.

The experiments at FNAL [17] and precision data from LEP [12] are compatible and give evidence for a heavy top quark with mass around  $m_t \simeq 170 - 180 \text{ GeV}$ . Such a heavy top has, to a good approximation, the property [18] that on average, it decays before it can form hadronic bound states. The Standard Model prediction is that the decay  $t \longrightarrow W b$  is predominant for a heavy top. Some information about its polarization and spin correlation may be preserved in its decay products. The spin effects can be analyzed through the angular correlation of the weak decay products. It is then possible to follow similar approaches as the ones followed in investigating the  $\tau$  dipole moments. However, at the high energies required for the  $t\bar{t}$  production, both the  $\gamma$  and  $Z$  electric and weak–electric dipole moments come into the game, and one has to define observables to disentangle each other.

A heavy top allows that the  $CP$  conjugates modes  $t_L\bar{t}_L$  and  $t_R\bar{t}_R$  are produced with a big percentage, contrary to low mass fermions. In [19, 20] an asymmetry sensitive to  $CP$ –violation constructed with the event rate difference of these modes was considered. This asymmetry can be measured through the energy spectra of prompt leptons coming from the decay channel  $t \longrightarrow W^+ b \longrightarrow l^+ \nu b$  and the conjugate one. The  $W^+$  is predominantly longitudinal, and assuming  $V - A$  weak interaction, the  $b$  quark is preferably left–handed. As the longitudinal  $W^+$  is collinear with the top polarization, so it is the  $l^+$  anti–lepton. Above the  $t\bar{t}$  threshold the top is produced with non zero momentum. As a result of the Lorentz boost a  $l^+$  coming from a  $t_R$  has a higher energy than the one produced in a  $t_L$  decay. The same happens in the conjugate channel and finally in the decay of the pair  $t_L\bar{t}_L$  the lepton from  $\bar{t}_L$  has a higher energy than the antilepton from  $t_L$ , while in the decay of  $t_R\bar{t}_R$  the anti–lepton has a higher energy. Therefore the energy asymmetry in the lepton is sensitive to the asymmetry

$$A = \frac{N(t_L\bar{t}_L) - N(t_R\bar{t}_R)}{N(t_L\bar{t}_L) + N(t_R\bar{t}_R)} \quad (30)$$

This asymmetry is  $CPT$ –odd and sensitive to the absorptive part of the electric and

weak-electric dipole moments.

To illustrate the size of possible CP-violating effects, a Weinberg model where the Higgs's matrix mixes  $CP$ -even and  $CP$ -odd scalars was considered:

$$\mathcal{L}_{\mathcal{Y}\cap\mathcal{H}\cap\mathcal{H}} = -\frac{\mathbb{I}_{\mathbb{H}}}{\mathbb{E}} \square (\mathcal{L} + \mathcal{L}^*) \square \quad (31)$$

For a reasonable combination of the mixing parameters, they estimated that for the Next Linear Collider at  $\sqrt{s} = 500 \text{ GeV}$   $m_t \simeq 150 \text{ GeV}$  and  $m_H \simeq 100 \text{ GeV}$ , the lepton asymmetry defined above is of the order  $10^{-3}$ . This value is accessible with  $10^7 \text{ } t\bar{t}$  pairs!

The up-down asymmetry in the azimuthal angular distribution, constructed from the rate difference between the events with  $l^\pm$  above and below the reaction plane is also a genuine  $CP$ -violating signal. This asymmetry is  $CPT$ -even and thus sensitive to the dispersive part of the electric and weak-electric dipole moments.

Similar ideas as the ones developed in [10] for the  $\tau$  lepton where also apply [21] to  $t\bar{t}$  production. Observables constructed from the momenta of the charged leptons and/or  $b$  jets originated from  $t$  and  $\bar{t}$  decay may be measurable in future experiments. These  $CP$ -odd and  $CPT$ -even observables result from the interference terms of the  $CP$ -even amplitudes with  $CP$ -odd ones, and are proportional to the dispersive part of the electric and weak-electric dipole moments. The absorptive parts can only contribute to the next order in the coupling constant, through the interference with absorptive parts of one-loop amplitudes. They also argue that possible  $CP$ -violating effects in  $t$  and  $\bar{t}$  decay do not contribute to leading order in perturbation theory. The one standard deviation accuracies obtainable for the dispersive and absorptive part of both electric and weak-electric dipole moments for the top decay channels  $t \longrightarrow bX_{had}$  and  $t \longrightarrow bl^+\nu_l$  is found to be close to  $10^{-17} \text{ e cm}$ , assuming 10.000  $t\bar{t}$  events with  $m_t = 150 \text{ GeV}$  at  $\sqrt{s} = 500 \text{ GeV}$ .

$CP$ -violating asymmetries were also studied [22] for the process  $e^+e^- \longrightarrow t\bar{t}$  with longitudinally polarized electrons. The work of [20] was extended in order to include polarization and to disentangle dispersive and absorptive parts of the electric and weak-electric dipole moments. With  $\sqrt{s} = 500 \text{ GeV}$ , an integrated luminosity of



$10fbn^{-1}$  and polarized electron beams with  $\pm 50\%$ ,  $90\%$  C.L. sensitivities of the order  $10^{-16} - 10^{-17} ecm$  are obtained. This is not enough to test many extended models [19, 20] that predicts dipole moments two or three orders of magnitude smaller.

Normal polarization to the production plane in  $e^+e^- \longrightarrow t\bar{t}$  was studied in [23]. In particular the one loop QCD correction was considered and the effect is induced from unitarity corrections; the electroweak contribution is less than the QCD one for a heavy top at  $\sqrt{s} = 500 GeV$ . In this reference it is also studied  $CP$ -violation in the top decay. Different normal polarization for  $t$  and  $\bar{t}$  is generated with a  $CP$ -violating lagrangean. In particular, the correlation of the azimuthal angles of the  $W^+$  and  $W^-$  is sensitive to this  $CP$ -violation.

## 5 Conclusions

We have discussed some topics related to the dipole moments of fermions. Their chirality-flip vertices may provide some insight into the origin of mass. Their property of being dimension-5 operators in the effective lagrangean suggest that (in conventional units) the anomalous magnetic weak-moment and the weak-electric moment are given by  $a_f^w \sim m_f^2/\Lambda^2$  and  $d_f^w \sim em_f/\Lambda^2$  respectively, where  $\Lambda$  is the scale of the physics involved.

Within the Standard Model, the weak-magnetic moment for the tau  $a_\tau^w(M_Z^2) = -(2.10 + 0.61 i) \times 10^{-6}$  receives its dominant contribution from the triangle loops with  $\nu WW$  and  $W\nu\nu$ .

There are observables in the process  $e^+e^- \longrightarrow \tau^+\tau^-$  which are linear in the weak dipole moments. In particular, the transverse (within the collision plane) and normal (to the collision plane) polarizations of single taus contain the information on  $a_\tau^w$  and  $d_\tau^w$ , respectively. These terms manifest themselves in  $\cos\Phi_h$  and  $\sin\Phi_h$  terms, respectively, of the angular distribution of the (hadron) decay product of the tau.

The weak-electric dipole moment appears in the P-odd, CP-odd spin correlation of both taus  $s_+^\tau s_-^\tau$ . This observable can be searched for in triple correlations for the momenta of the decay products. At LEP, the OPAL and ALEPH experiments have

used this method to put the bounds  $|d_\tau^e| \leq 10^{-17} e\text{ cm}$ . On the other hand, the P-even, T-odd normal polarization of single taus has the virtue to involve the axial (instead of vector) coupling of the electron.

The electric and weak-electric CP-odd dipole moments for the  $t$ -quark can be searched for by means of CP-odd observables at the Next Linear Collider.

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